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Abstracts

戦略的創造研究推進事業
Core Research for Evolutional Science and Technology

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Sharp Interface Limits for Two-Phase Flows

We will discuss the limits of diffuse interface models for a two-phase flow of incompressible, viscous fluids when the parameter related to the interfacial thinks tends to zero. So far such limits have been derived formally and on the level of so-called varifold solutions, which is a rather weak notion of solution for the sharp interface model. We will present a recent result on the rigorous analysis of this limit for short times in the case of a simple model for a two-phase flow with phase transition, which consists of a Allen-Cahn-Stokes system.

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Giuseppe Da Prato
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Some results on Kolmogorov equations for stochastic Navier–Stokes equations

We are concerned with a $2D$ or $3D$ Navier–Stokes equation

$$\begin{cases} dX = (AX + b(X))dt + \sqrt{Q}\, dW(t), \\ \\ u(0,x) = \varphi(x), \end{cases} \tag{1}$$

in an open subset $\mathcal{O}$ of $\mathbb{R}^d$ ($d = 2$ or 3) with suitable boundary conditions. Here A is the Stokes operator in the space of divergence free functions of $L^2(\mathcal{O};\mathbb{R}^d)$ (P denotes the projector on this space) , $b(x) = P(x \cdot \nabla x))$ and $\sqrt{Q}\, W$ a noise perturbation.

The corresponding Kolmogorov equation looks like

$$\begin{cases} u_t(t,x) = \frac{1}{2}\ \mathrm{Tr}\ [QD_x^2(t,x)] + \langle Ax + b(x), D_x u(t,x)\rangle = \mathcal{L}u(t,x), \\ \\ u(0,x) = \varphi(x), \end{cases} \tag{2}$$

where $\mathcal{L}$ is defined on a suitable space of test function $\mathcal{E}$.

Formally the relationship between equations (1) and (2) is given by

$$u(t,x) = \mathbb{E}[\varphi(X(t,x))], \quad t \geq 0,\ x \in H. \tag{3}$$

In the first part of the talk we present a result of existence and uniqueness of a solution of (2) in $L^2(H,\nu)$ (ν being an invariant measure) when $d = 2$ and moreover that $\mathcal{E}$ is a core for $\mathcal{L}$ (paper in collaboration with V. Barbu and A. Debussche). We deduce several consequences of this fact concerning the definition and some properties of the Sobolev spaces $W_Q(H,\nu)$.

In the second part we consider the case $d = 3$ and prove the existence but not the uniqueness of a regular solution of (2), provided φ is regular enough (paper in collaboration with A. Debussche).

We show different attempts to exploit this result to get the uniquenesss in law for 3D-NS, unfortunately unsuccsessfully so far.

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Irina Vlad. Denisova

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On energy inequality for the problem on two-phase capillary fluid motion in Oberbeck-Boussinesq approximation

We consider the Oberbeck–Boussinesq approximation for unsteady motion of a drop in another fluid. On the unknown interface between the liquids, the surface tension is taken into account. We study this problem in Hölder classes of functions, classical existence theorem for the problem was proved for a finite time in [1]. Basing on the idea of constructing a function of generalized energy for the fluid proposed by M. Padula in [2], we show that the L_2–norms of the velocity and deviation of the temperature from the mean value decay exponentially with respect to time. The proof uses a similar result without including temperature obtained in the joint paper of the author and V. A. Solonnikov [3]. Moreover, there was proved global solvability of the problem with constant temperature.

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Eduard Feireisl

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Weak solution to problems involving perfect fluids

The dynamics of a perfect (inviscid) fluid in the physically relevant 3D setting features phenomena that are still very poorly understood. The recent ground breaking results of C. De Lellis and L. Székelyhidi revealed very surprising properties of the weak solution of the (incompressible) Euler system:

- for any initial data there exist infinitely global-in-time weak solutions;
- there exists a family of bounded initial data for which the system admits infinitely many globalin- time solutions conserving the mechanical energy, as a matter of fact, the energy may behave in an arbitrary way;
- these wild phenomena persist even in the class of Hölder solutions with certain exponent.

We discuss to which extent these properties are contagious and may be shown for solutions of systems involving the equations of a perfect fluid as a component. To this end, we study robustness of a given system with respect to velocity oscillations and identify a couple of specific examples, among which:

- Euler-Fourier system describing the motion of an inviscid but still heat conducting fluid;
- Euler-Korteweg system and equations of quantum hydrodynamics;
- Euler-Cahn-Hilliard and related system arising in the phase field models of mixtures.

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Yasuhide Fukumoto

Institute of Mathematics for Industry, Kyushu University, Fukuoka, Japan

Local and global analyses of azimuthal magnetorotational instability

Magnetorotational instability (MRI) discovered by Velikhov (1959) and Chandrasekar (1960) has attracted attention since Balbus and Hawley (1991) pointed out that the MRI can trigger turbulence necessary to account for outwards transport of the angular momentum, while forming a star in the center, in the accretion disks.

Short-wavelength stability analysis is made of axisymmetric rotating flows of an electrically conducting fluid subjected to external azimuthal (=toroidal) magnetic field. This instability of the MHD is referred to as the azimuthal magnetorotational instability (AMRI), Non- axisymmetric perturbations, when coupled to azimuthal magnetic field, makes unstable rotating flows of a wide variety of angular-velocity and magnetic-field profiles. We determine the range of the unstable profiles and the overall maximum growth rate for the AMRI.

Then a global modal analysis is made for limited case of a rigid-body rotation subjected to the azimuthal magnetic field with its strength proportional to the distance from the axis of symmetry. We derive formulas for the MHD waves and highlight the viewpoint of the Hamiltonian spectra by calculating energy of waves.

(This is a joint work with Rong Zou.)

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Tadahisa Funaki

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Choosing a proper minimizer of a certain variational problem from microscopic viewpoint

We discuss a variational problem whose minimizer is a solution of a free boundary problem. This is studied by Alt, Caffarelli, Friedman, Weiss and others. We are especially interested in the situation that several minimizers exist. Such variational problem can be derived from microscopic systems such as Ising model or -interface model. Microscopic systems have more detailed structure than the macroscopic variational problem, so that a careful study of microscopic systems enables us to pick up a proper minimizer from several candidates. The talk is based on joint works with E. Bolthausen, T. Chiyonobu and T. Otobe.

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Giovanni Paolo Galdi

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On the motion of a rigid body with a liquid-filled cavity

Let S be the coupled system constituted by a rigid body with an interior cavity completely filled with a viscous liquid. In this talk I shall present a number of analytical results concerning the motion of S, with or without the action of external forces. In particular, I show that the presence or absence of the liquid can dramatically affect the ultimate dynamics of S. I will also present suitable numerical tests that, on the one hand, corroborate the mathematical analysis and, on the other hand, pose new questions that lay the foundations for future analytic work.

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Toshiaki Hishida

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Asymptotic structure of steady viscous incompressible flow around a rotating obstacle in 2D

Let us consider the motion of a viscous incompressible fluid around an obstacle (rigid body) in the plane $\mathbb{R}^2$. As compared with 3D problem, we have less knowledge about exterior steady flows in 2D despite efforts of several authors. The difficulty is to analyze the asymptotic behavior of the flow at spatial infinity. This is related to the hydrodynamical paradox found by Stokes. In this talk I would like to revisit the plane steady Stokes flow when the obstacle is rotating with constant angular velocity. Due to the rotating effect, there is no longer Stokes' paradox. In fact, the flow can be bounded at infinity even if the net force does not vanish. Our analysis relies on precise asymptotic structure of the fundamental solution. The centering technique is often employed to justify several calculations. We construct the Stokes flow that decays to zero as $|x| \to \infty$ so long as the external force vanishes at large distance.

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Makoto Iima

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Analysis of the dynamics of localized convection patterns

Intrinsic nature of dynamics of coherent structures is closely related to the hidden mathematical structure of the systems. Even if the governing equation is given, such mathematical structure can not always be visible in the numerical simulation of time evolution. In this talk, through various approaches to three topics, we show how the idea of “mathematical structures” was used to analyze dynamics of coherent structures.

First, we show how relevant solutions and their bifurcation structure is used to analyze the dynamics of the pattern formation problems in binary fluid convection. We succeeded in obtaining a time-periodic traveling wave solution which represents the localized traveling wave, which has been been observed in experiments and time-evolution of numerical simulations. Bifurcation structures can be used in the collision problems and in the pattern formation process.

Second, we show how insights of mathematical structure can be used in the experiments of localized bioconvection cells. After commenting the importance of initial states, we show interesting localized convection patterns and their interactions, which are similar to ones in binary fluid convection. Then we show several experimental results suggesting the bistability.

Third, we show how the orbit representing the dynamics can be analyzed. A well-known self-replicating phenomena observed in a reaction-diffusion system was analyzed in terms of the dynamics of the perturbations to the orbit, which are classified by using the eigenvectors of the final (or initial) state.

The topics of this talk are joint works with Dr. T. Yamaguchi (Hiroshima University), Ms. E. Shoji (Hiroshima University), Dr. T. Watanabe (JAXA) and Prof. Y. Nishiura (Tohoku University).

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Hideo Kozono

Department of Mathematics, Waseda University, Tokyo, Japan

Stability of the large stationary solutions to the Navier-Stokes equations under the general flux condition

Consider weak solutions of the stationary Navier-Stokes equations in a 3D bounded domain with the non-homogeneous boundary condition. We give a new criterion on stability of stationary flows under an L^2-initial disturbance of the non-stationary flow with an exponential convergence rate. Our result may be regarded as a concrete characterization of the stability condition which was originally given by Sattinger. It should be noted that our stability condition necessarily needs neither smallness of stationary flows nor that of initial disturbance. Furthermore, some examples of large stable solutions are given.

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Takeshi Miyazaki
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Statistical Mechanics of Quasi-geostrophic Vortices

The statistical mechanics of quasi-geostrophic vortices is investigated numerically and theoretically. Direct numerical simulations of a point vortex system of mixed sign under periodic boundary conditions are performed using a fast special-purpose computer for molecular dynamics (GRAPE9), whose results are compared with those from pseud-spectral simulations of decaying quasi-geostrophic turbulence.[1, 2] Clustering of point vortices of like sign is observed and two tall columnar vortices appear as equilibrium states, even in the Hamiltonian dynamical system of point vortices.

In order to explain the numerical results, a three-dimensional mean field equation is derived based on the maximum entropy theory.[3] The numerically obtained end states are shown to be the two-dimensional sn-sn dipole solutions of the sinh-Poisson equation (i.e., two-dimensional mean field equation). We present other branches of two- and three-dimensional solution of the three-dimensional mean field equation. The entropy of these solution branches is found to be smaller than that of the two-dimensional sn-sn dipole branch. They are known to be extremum entropy states. The stability of the maximum and extremum entropy states is studied theoretically and numerically. The two-dimensional (sn-sn dipole and zonal) solutions are stable against disturbances of finite amplitude, whereas the three-dimensional solutions are shown to be unstable. These findings explain the reason why only the two-dimensional sn-sn dipole states are found in the numerical simulations of point vortices. Also, the influence of the aspect ratio of the periodic box is discussed.

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Takaaki Nishida
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Heat convection problems of compressible viscous fluids

Heat convection problems of compressible, viscous and heat-conducting fluids are discussed. The fluids occupy a horizontal strip domain $z_0 < z < z_0 + 1$ heated from below $(z = z_0 + 1)$ under gravity directing z-direction with periodic boundary condition in horizontal directions. We use the non-dimensional parameter following Spiegel :

$$L = z_0 + \tfrac{1}{2} = \tfrac{T_u+T_l}{2\beta d}, \quad \beta = \tfrac{T_l-T_u}{d} \text{ is temperature gradient.}$$

Stationary solutions and stationary bifurcations are considered around the equilibrium state. Bifurcating solutions (pattern formations) are obtained for $\mathcal{R}_m \geq \mathcal{R}_c(L)$ uniformly with respect to $L \geq L_1$, where $\mathcal{R}_m$ is the Rayleigh-Spiegel number.

The limit of $L \to +\infty$ corresponds to those of Oberbeck-Boussinesq equations (incompressible system), when the critical Rayleigh-Spiegel number $\mathcal{R}_c(L)$ converges to $\mathcal{R}_c(+\infty)$ of the critical Rayleigh number of Oberbeck-Boussinesq equations.

The time evolution problem is also formulated and the linearized system is discussed globally in time and uniformly with respect to L and also about limit of $L \to \infty$.

(This is a joint work with Mariarosaria Padula and Yoshiaki Teramoto (Setsunan University).)

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Hirofumi Notsu

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A stabilized Galerkin-characteristics finite element scheme for the Navier-Stokes equations: theory, computations and applications

A stabilized Galerkin-characteristics finite element scheme for the Navier-Stokes equations with theoretical and numerical results and its applications are presented. It is a combined finite element scheme with a Galerkin-characteristics method and Brezzi-Pitkäranta's pressure stabilization. The scheme is symmetric and robust for high Reynolds number problems and employs a cheap P1/P1-element which yields a small number of degrees of freedom. The scheme is, therefore, efficient especially for three-dimensional problems. Stability and convergence with error estimates are proved and two- and three-dimensional numerical results are shown. Furthermore, applications of the scheme to some flow problems, e.g., a problem with a time-dependent domain, are exhibited.

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Toshiyuki OGAWA
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Stabilization of unstable patterns in reaction-diffusion systems

Oscillatory patterns are known to bifurcate in RD systems as well as stationary patterns. The purpose of this talk is to show the possibility to effectively generate various desired spatial oscillating patterns by guiding such patterns suitably. We first introduce the 3-component RD system which exhibits wave instability and prepare feedback stabilization control in terms of spatial spectrum consensus so that we can stabilize unstable standing waves. This talk is based on the joint work between Kenji Kashima (Kyoto University) and Yusuke Umezu (Osaka University).

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Issei Oikawa
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A discrete gradient method for the Rayleigh-Plesset-Keller equation

1 Introduction

This paper is concerned with a discrete gradient method for the Rayleigh-Plesset-Keller equation. The Rayleigh-Plesset(RP) equation is proposed by Rayleigh and Plesset[4, 3] to describe the motion of a spherical single bubble filled with a vapor and non-condensable gas in a liquid. The RP equation is a second-order nonlinear ordinary differential equation derived from the incompressible Navier-Stokes equations with spherical symmetry:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = -\frac{1}{\rho}\left(p_\infty - p_B(R) + \frac{2\sigma}{R}\right). \tag{1.1}$$

[1]This is a joint work with M. Ohnawa(Waseda Univ.) and Y. Suzuki(Waseda Univ.).

Here $R = R(t)$ is the radius of the bubble(unknown), ρ is the density of the liquid, σ is the surface tension, p_∞ is a given external pressure and $p_B(R)$ is the pressure inside the bubble. The pressure $p_B(R)$ is given by

$$p_B(R) = p_V + \bar{p}_G \left(\frac{R_0}{R}\right)^{3\kappa},$$

where p_V is the vapor pressure, R_0 is the reference radius, κ is the heat capacity and $\bar{p}_G$ is defined by $\bar{p} - p_V + 2\sigma/R_0$ with a given reference pressure $\bar{p}$.

In [2], Keller and Kolodner proposed the modified PR equation by taking the effect of the compressibility of a liquid into account, which is called the Rayleigh-Plesse-Keller equation or the Keller equation for short. The Keller equation reads as

$$\left(1 - \frac{\dot{R}}{c}\right) R\ddot{R} + \frac{3}{2}\left(1 - \frac{\dot{R}}{3c}\right)\dot{R}^2 = -\frac{1}{\rho}\left(1 + \frac{\dot{R}}{c} + \frac{R}{c}\frac{d}{dt}\right)\left(p_\infty - p_B(R) + \frac{2\sigma}{R}\right), \tag{1.2}$$

where c is the sound speed. The Keller equation describes the radiation dumping of the oscillating bubble, whereas the RP equation does not. It is known that the Keller equation provides a good approximation up to a Mach number of 0.3.

For mathematical and numerical analysis of these equations, we show that the RP and Keller equations can be rewritten into gradient systems. As a result, we see that the inviscid RP equation has a Hamiltonian structure, and the Keller equations have an energy-dissipative property. Furthermore, energy-preserving or -dissipative schemes are deduced by applying the discrete gradient method to the gradient systems.

2 Gradient systems

We introduce a canonical momentum $Q = \rho R^3 \dot{R}$ and $P(R) = p_\infty - p_B(R) + \frac{2\sigma}{R}$. Let $(R_*, 0)$ be the equilibrium point of the RP equation. The energy function is defined by

$$H(R, Q) = \frac{Q^2}{2\rho R^3} + \int_{R_*}^{R} P(S) S^2 dS.$$

The RP equation can be rewritten into a gradient system with the energy function:

$$\begin{pmatrix} \dot{R} \\ \dot{Q} \end{pmatrix} = \mathbf{A}(R, Q)\nabla H := \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \partial HR \\ \partial HQ \end{pmatrix}. \tag{2.1}$$

The matrix $\mathbf{A}(R, Q)$ is negative semi-definite, which yields $\frac{d}{dt} H(R(t), Q(t)) = 0$.

With the same energy function H, the Keller equation is also rewritten as

$$\begin{pmatrix} \dot{R} \\ \dot{Q} \end{pmatrix} = \mathbf{A}_K(R, Q)\nabla H := \begin{pmatrix} 0 & 1 \\ -1 & \alpha(R, Q) \end{pmatrix} \begin{pmatrix} \partial HR \\ \partial HQ \end{pmatrix}, \tag{2.2}$$

where α is a certain function which is negative if $\dot{R}(t) < c$. Hence we have $\frac{d}{dt} H(R(t), Q(t)) \leq 0$ until $\dot{R}(t) < c$.

3 Discrete gradient method

The discrete gradient method is a class of energy-preserving or structure-preserving numerical methods using the discrete gradient operator, and is already well-studied. Applying the discrete gradient method to the gradient systems, we can obtain an energy-preserving or -dissipative scheme. We here employ the coordinate increment method proposed by Itoh and Abe [1]. The scheme is given by

$$\begin{pmatrix} \dfrac{R_{n+1}-R_n}{h} \\ \dfrac{Q_{n+1}-Q_n}{h} \end{pmatrix} = \mathbf{A}(R_n, Q_n) \begin{pmatrix} \dfrac{H(R_{n+1}, Q_{n+1}) - H(R_n, Q_{n+1})}{R_{n+1}-R_n} \\ \dfrac{H(R_n, Q_{n+1}) - H(R_n, Q_n)}{Q_{n+1}-Q_n} \end{pmatrix},$$

where h is the time-step size. We provide a global discretization error estimate for the discrete gradient scheme. Numerical results are presented to verify the validity of the discrete gradient scheme for the RP and Keller equations.

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Hisashi Okamoto

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Topologically simple patterns appearing in the 2D Navier-Stokes equations at large Reynolds numbers

We study stability and bifurcation of stationary and time-periodic solutions of Kolmogorov's problem for the Navier-Stokes equations in 2D flat tori. Specifically we look for a unimodal solution, which is characterized by having a large, topologically simple pattern of streamlines. We present a conjecture that such simple patterns emerge in steady-states or time-periodic solutions at large Reynolds numbers, no matter what the external force may be. We confirm this conjecture by some numerical experiments. Thus the well-noted fact that a large structure appears in 2D large Reynolds number flows is reinforced in another form.

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Linyu Peng

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Variational bicomplexes, multisymplectic structures and applications

Multisymplectic partial differential equations have a sound property of structure preservation, the conservation of multisymplecticity. Multisymplectic integrators are numerical methods preserving multisymplecticity. We show that the variational bicomplexes provide systematic approaches for understanding the conservations of multisymplecticity for both differential equations and variational integrators. Illustrative examples from fluid dynamics and physics are provided.

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Jan Prüss

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Thermodynamically consistent models for nematic liquid crystal flows

We consider the non-isothermic Erickson-Leslie model for nematic liquid crystal flows, discuss its thermodynamic consistency, and prove local well-posedness in the setting of L_p-spaces. We further identify the equilibria, show that they are local maxima of the entropy, and prove their asymptotic stability with phase by means of the generalized principle of linearized stability.

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Tudor Ratiu

Section of Mathematics, Swiss Federal Institute of Technology in Lausanne, Switzerland

Geometric formulation of nematodynamics

The main two theories for conservative nematic liquid crystals (the Ericksen-Leslie and Eringen) will be presented from a geometric variational point of view. The precise conditions ensuring that the Ericksen-Leslie theory is included in the Eringen approach will be discussed. Taking advantage of the newly found geometric approach used in solving this longstanding problem, a well-posedness result for the dissipative Ericksen-Leslie equations will be formulated.

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Takashi Sakajo

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On point vortex-α system and vortex collapse with enstrophy dissipation

Holm et al.[3, 4] proposed the Euler-α equations for incompressible flows $\boldsymbol{u}(t,\boldsymbol{x})$ in space $\boldsymbol{x} \in \mathbb{R}^3$ and time t as follows.

$$(1-\alpha^2\Delta)\partial_t\boldsymbol{u}+\boldsymbol{u}\cdot\nabla(1-\alpha^2\Delta)\boldsymbol{u}-\alpha^2(\nabla\boldsymbol{u})^T\cdot\Delta\boldsymbol{u} = -\nabla p, \quad \nabla\cdot\boldsymbol{u}=0, \quad \boldsymbol{u}(\boldsymbol{x},0)=\boldsymbol{u}_0(\boldsymbol{x}). \tag{3.1}$$

The Euler-α equations are a dispersive regularization of the Euler equations[4]. The first momentum equation is obtained by taking an average of the spatial flow information below the small scale α. When the Euler-α equations are considered in $\mathbb{R}^2$, Linshiz and Titi[8] have shown that there exists a unique global solution of (3.1) for the initial velocity field in the Sobolev space $H^m(\mathbb{R}^2)$, $m > 2$ and that the solution converges to that of the Euler equations in $L^\infty([0,\infty);H^m)$.

When we define the scalar α-vorticity as $q = (1-\alpha^2\Delta)\nabla^\perp\boldsymbol{u}$, the 2D Euler-$\alpha$ equations are reduced to the following transport equation for q:

$$q_t + (\boldsymbol{u}\cdot\nabla)q = 0, \qquad \boldsymbol{u} = K^\alpha * q, \qquad q(\boldsymbol{x},0) = (1-\alpha^2\Delta)\nabla^\perp\cdot\boldsymbol{u}_0(\boldsymbol{x}). \tag{3.2}$$

Here, the kernel $K^\alpha(\boldsymbol{x})$ is given by $K^\alpha(\boldsymbol{x}) = -\frac{1}{2\pi\alpha^2}K_0\left(\frac{|\boldsymbol{x}|}{\alpha}\right) * \frac{1}{2\pi}\nabla^\perp\log|\boldsymbol{x}|$, where $K_0(\boldsymbol{x})$ is a modified Bessel function of the second kind[12]. Suppose now that the α-vorticity belongs to the space of Radon measure on $\mathbb{R}^2$ whose support consists of N discrete points $\boldsymbol{x}_m(t) = (x_m(t), y_m(t))$ for $m = 1,\dots,N$, namely,

$$q(t,\boldsymbol{x}) = \sum_{m=1}^{N}\Gamma_m\delta(\boldsymbol{x}-\boldsymbol{x}_m(t)). \tag{3.3}$$

Then the equation (3.2) gives rise to the ordinary differential equations for the N point singularities as follows.

$$\frac{\mathrm{d}x_m}{\mathrm{d}t} = -\frac{1}{2\pi}\sum_{m\neq n}^{N}\Gamma_m\frac{y_n-y_m}{l_{mn}^2}B_K\left(\frac{l_{mn}}{\alpha}\right), \quad \frac{\mathrm{d}y_m}{\mathrm{d}t} = \frac{1}{2\pi}\sum_{m\neq n}^{N}\Gamma_m\frac{x_n-x_m}{l_{mn}^2}B_K\left(\frac{l_{mn}}{\alpha}\right), \tag{3.4}$$

where $l_{mn} = |\boldsymbol{x}_m - \boldsymbol{x}_n|$, $B_K(x) = 1 - xK_1(x)$ with $K_1(x)$ being another modified Bessel function of the second kind[12]. The equation (3.4) is called *the point-vortex-α (PV-α) system.*

The PV-α system has been considered to construct a singular incompressible flow that shares a common property with 2D turbulent flows[11], whose theoretical background is explained as follows. According to [1, 6, 7], there emerges the inertial range in the energy density spectrum of the 2D turbulent flows for sufficiently small viscosity. This phenomenon is induced by the enstrophy dissipation in the inviscid limit of the incompressible flows, but the enstrophy is conserved by smooth solutions of the 2D Euler equations. Accordingly, the 2D turbulent flows should be characterized as non-smooth singular weak solutions of the 2D Euler equations that dissipate the enstrophy. Mathematically, Eyink[2] has shown that weak solutions of the 2D Euler equations in $L^p(\mathbb{R}^2)$ ($p > 2$) can not dissipate the enstrophy in a weak sense. This indicates that it is necessary for us to deal with the initial vorticity data in a weaker space, the space of Radon measure on $\mathbb{R}^2$ for instance, to obtain such weak solutions with the anomalous enstrophy dissipation.

Assuming that the support of vorticity distribution is represented by N discrete points as in the PV-α system, we can reduce the 2D Euler equations to the ODEs for these N point singularities. This system is referred to as the *point vortex (PV) system*, which has been investigated as mathematical models of coherent vortex structures. Many references are found in [9]. Among them, Kimura[5] has shown that the PV system has a singular solution, called *self-similar collapse*, in which the point vortices collapse self-similarly to a point in finite time. However, the self-similar collapse is not an appropriate candidate of the singular solution with the enstrophy dissipation, since the solution of the PV system is not a weak solution of the 2D Euler equations. On the other hand, when the PV-α system with $N = 3$ is considered for the same initial data as Kimura's, it has been shown that the solution of (3.4) exists globally in time for $\alpha \neq 0$, while the global solution converges to the self-similar collapse followed by the self-similar expanding solution beyond the critical time as $\alpha \to 0$[11]. Furthermore, the singular solution dissipates the enstrophy at the critical time in the sense of distribution.

Although the singular solution has an desirable property in 2D turbulent flows, it is mathematically uncertain whether this singular solution is a weak solution of the 2D Euler equations with the anomalous enstrophy dissipation. Physically, we are not sure if there exist more singular solutions with the enstrophy dissipation defined as the $\alpha \to 0$ limit of the PV-α system. In the present talk, we are going to consider these problems. First, according to [10], the Euler-α equations have a unique global weak solution for the initial vorticity data in the space of Radon measure on $\mathbb{R}^2$ in the sense of distribution. We confirm that the solution of the PV-α system gives rise to a weak solution of the Euler-α equations. Second, we investigate the existence of the other singular solutions that dissipate the enstrophy for the PV-α system for $N \geq 4$ by numerical means. This study is a joint work with Takeshi Gotoda and it is partially supported by the Grant-in-Aid for Scientific Research (B) #26287023.

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Nonlinear surface waves on the plasma-vacuum interface

In our talk we discuss the propagation of weakly nonlinear surface waves on a plasma-vacuum interface. In the plasma region we consider the equations of incompressible magnetohydrodynamics, while in vacuum the magnetic and electric fields are governed by the Maxwell equations. Under a physical condition that makes the problem linearly weakly stable, surface wave propagate along the plasma-vacuum interface.

Following Hunter's approach, we measure the amplitude of the surface wave by the normalized displacement of the interface in a reference frame moving with the linearized phase velocity of the wave, and obtain that it satisfies an asymptotic nonlocal, Hamiltonian evolution equation. We show the local-in-time existence of smooth solutions to the Cauchy problem for the amplitude equation in noncanonical variables, and we derive a blow up criterion.

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Mathematical modeling of atherosclerosis

Atherosclerosis, the major cause of cardiovascular disease, is a chronic inflammation that starts when LDL (low-density proteins) cholesterol enter the intima of the blood vessel where they are oxidized. The anti-inflammatory response of oxLDL triggers the response of monocytes that are transformed into macrophages and foam cells, leading to the production of inflammatory cytokins and further recruitment of monocytes. This complex process leads to the formation of an atherosclerotic plaque (atherogenesis) and possibly to its rupture.

Several theories have been developed to explain the pathogenesis of atherosclerosis but none of them can explain the whole process due to the large number of factors involved. On the other hand, mathematical models should account for these complex multiphysics phenomena. They are described by nonlinear reaction- diffusion equations, coupled with fluid and structure equations and only a few results exist for some simplified models.

In this talk we study the well-posedness of both one-dimensional and two-dimensional simplified models capturing essential features of the early stage of atherosclerosis development.

This work has been done in collaboration with T. Silva and J. Tiago.

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Vsevolod Solonnikov

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On free boundary problems of hydrodynamics and magnetohydrodynamics

The communication is devoted to the solvability of some free boundary problems of hydrodynamics for viscous fluids. The special attention is given to the global solvability results (when the initial data are close to the equilibrium states).

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Hiroshi Suito

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Blood flow structure in the aorta and its relation to geometrical characteristics

In this paper, we present a fluid–structure interaction analysis of blood flow in the thoracic aorta as it relates to aortic aneurysms. Thoracic aortic aneurysm is one of the life-threatening diseases, which slowly grows with advancing age of the patient and may be at a risk of rupture. Many papers have reported on the risk factors of rupture, however, natural history of the development of an aneurysm has not been fully understood [2].

There are so many parameters characterizing the blood flow, for example, geometric, kinematic and physiologic parameters. The information on which parameter would be the most important to predict for the generation and development of the aneurysms should be useful for clinical medicine. In this study, we draw attention to geometrical characteristics of the blood vessels. Differences in the geometry of the blood vessels bring about differences in the flow characteristics, which cause different wall shear stresses (WSS) and Oscillatory Shear Index (OSI) distributions.

We consider a number of patient-specific models of the aorta as constructed from CT scans. We compute the flow field with the variational multiscale version of the Deforming-Spatial-Domain/Stabilized Space–Time method (DSD/SST-VMST) [1, 3, 4].

This work was supported by JST-CREST Mathematics program.

(This is a joint work with K. Takizawa, V. Q. Huynh, T. Ueda and T. E. Tezduyar)

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Classical Solvability of the Two-Phase Radial Viscous Fingering Problem in a Hele-Shaw Cell

Classical Stefan problem is a nonlinear problem with a free boundary for heat equation. Concerning a problem with analogous conditions on the free boundary for elliptic equations, one arrives at the two-phase Hele-Shaw problem (the Muskat or Muskat-Leibenzon problem), which is of hydrodynamic type: in this case the time dependency of the problem is preserved due to the variation of the free boundary with time.

In this communication, we are concerned with proving the classical solvability of the two-phase Hele-Shaw problem with radial geometry by applying the same method as in the Stefan problem and justifying the vanishing the coefficients of the derivatives with respect to time in parabolic equations.

(This is a joint work with H. Tani)

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Katsuhiro Yamamoto

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Investigation of Bubble Clouds in a Cavitating Jet

The water jet technology is well known as a technique to make effective use of the damage potential of cavitation. Especially, high-speed submerged water jet is composed of large number of cavitation clouds with a high erosive property. In this study, the properties of cloud cavitation in the high-speed water jet are investigated by experimental observation and numerical simulations.

In the experiment, a high-speed video camera at frame rate of about 5×10^5 fps is used to observe the unsteady behavior of free high-speed cavitating jets which are ejected from a narrow nozzle into still water, and the behavior of the generated bubble clouds. It is found that high-pressure pulses are formed by collapsing bubble clouds, and that the pulses start a few microseconds before the collapse. The propagation speeds of the pressure pulses are about 1000 m/s, and the incidence frequencies are within the audible range. The erosion test due to the impingement of the water jet to aluminum specimen shows that the mass loss curves have two peaks. The second peak in the curve come mainly from cloud cavitation, because the second peak is some distance from nozzle outlet and the amount of mass loss is weekly dependent on the injection pressure.

In order to explain the experimental results, two cavitation models are employed. The first is a simplified continuum model of a homogeneous two-phase flow, and the other is a spherical cloud model filled with the cavitation bubbles. The intermittent generation of the cavitating jets is numerically simulated by the first model, and the focusing of a spherical wave at the center of the cloud is computed by the second model. Homogeneous two-phase flow in both model is highly compressible and the changes of sound speeds are very large, therefore the discontinuous changes can be formed easily in the flow. For such flow the finite difference scheme has to be stable and highly accurate under the restriction of CFL condition. For this reason, CIP method which is a kind of the time splitting method is chosen in this study. The computational result shows that the inward spherical wave produced a large pressure pulse propagating to a pure water, and the small bubbles in the cloud intensify the pressure increase. However the calculated value of the rebound period of bubble cloud is too short compared with the experimental values.

In summary, this paper has described that the cavitation clouds can generate higher impulsive pressure with high frequency than individual bubble, and it has a possibility of severe damage on material surface. While, some problems in the numerical simulation are also identified.

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Unsteady Behaviors of Cavitation Bubbles, Clouds and Induced Shock Waves

Much effort has been made to model unsteady phenomena in cavitation and multiphase flow dynamics such as rebound behavior of growth-collapse of cavitation bubbles and clouds, which induce shock waves propagating into surrounding bubbly liquids. In this talk, we will first show some experimental results of single bubble dynamics and will clarify that there exists a perturbation of the radial velocity of a bubble in the rebound behavior, in which the perturbation may be modeled by a Wiener process. Then we will develop stochastic Rayleigh-Plesset equations for the single bubble dynamics with noise. Second, we will study the unsteady behavior of a cavitation cloud, in which we can observe a rebound behavior similar to the single bubble dynamics and also observe shock wave propagation induced from collapse of the cavitation cloud. Finally, we will propose a mathematical model to elucidate the unsteady behavior of the rebound of a cavitation cloud with the shock wave propagation induced from the collapse.

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