Syllabus

## 1. Pattern formations in fluids and computer assisted analysis.

**(Takaaki Nishida)**

1.1 Examples of pattern formations in fluids.

1.2 Bifurcation theories.

1.3 Applications of bifurcation theorems.

1.4 Computer assisted analysis.

1.5 Computer assisted analysis in mathematical fluid mechanics.

Course Description

Pattern formations from the equilibrium state in fluid motions may be treated by the bifurcation theories. Bifurcation theorems can be applied to explain Taylor vortices of Taylor problems and hexagonal cells of heat convection problems as the first bifurcation. Computer assisted analyses become necesssary to see global bifurcation structures. Examples of computer assisted proofs and anlyses are explained for some heat convection problems.

## 2. Introduction to the theory of stochastic differential equations and stochastic partial differential equations.

**(Tadahisa Funaki)**

2.1. Some basic concepts in probability theory.

2.2. Brownian motion.

2.3. Stochastic integrals and Ito's formula.

2.4. Stochastic differential equations.

2.5. Stochastic partial differential equations.

Course Description

After briefly summarizing in Section 1 the basic concepts and facts in probability theory such as probability spaces, random variables, their convergences, independence, the central limit theorem and Gaussian distributions, the Brownian motion will be introduced in Section 2. The stochastic integrals are essential to discuss stochastic (partial) differential equations and Ito's formula plays a central role in the calculus related to them. After these preparations, the stochastic differential equations and the stochastic partial differential equations will be discussed in Sections 4 and 5, respectively. If time permits, as an example of the stochastic partial differential equation, I will talk about the singular limit for stochastic reaction diffusion equations.

## 3. $L^r$ Helmholtz decomposition and its application to the Navier-Stokes equations. (Hideo Kozono)

**3.1** Helmholtz-Weyl dcomposition in Lr.

**3.2** Lr-variational inequality.

**3.3** Stationary Navier-Stokes equations under the general flux condition.

**3.4** Global Div—Curl lemma.

**3.5** General compensated compactness theorem.

Course Description

We show that every $L^r$ -vector field on D can be uniquely decomposed into two spaces with scalar and vector potentials and the harmonic vector space via rot and div, where D is a bounded domain in $R^3$. This may be regarded as generalization of de Rham—Hodge decomposition for smooth k-forms on compact Riemannian manifolds. Our result holds not only smooth but also general $L^r$ vector fields. Basically, construction of harmonic vector fields is established by means of the theory of elliptic PDE system of boundary value problems due to Agmon—Douglis—Nirenberg. Since we deal with $L^r$ —vector fields, such a general theory is not directly available. To get around this difficulty, we make use of certain variational inequalities associated with the quadtatic forms defined by rot and div. various kinds of boundary conditions which are compatible to rot and div and which determine the harmonic parts are fully discussed.

As applications, we first consider the stationary problem of the Navier—Stokes equations in multi-connected domains under the inhomogeneous boundary condition. Up to the present, it is na open question whether there exists a solution if the given boundary data satisfies the general flux condition. It will be clarified that if the harmonic extension of the boundary data into D is small in $L^3(D)$ compared with the viscosity constant, then there is at least one weak solution.

The second application is on the global Div—Curl lemma. The classical Div—Curl lemma is stated in such a way that the convergence holds in the sense of distributions. Under the boundary condition determining the harmonic vector fields In the $L^r$ —Helmholtz—Weyl decomposition in D, we show that the convergence to holds in the whole domain D. Furthermore, we give a more general compensated compacteness theorem in the Hilbert space associated with the global Div—Curl lemma.

## 4. Real interpolation and endpoint estimates for the Stokes semigroup. (Masao Yamazaki)

4.1 Definition, fundamental property and concrete examples of real interpolation.
4.2 Dual spaces and real interpolation.
4.3 The $L^q - L^r$ estimates for the Stokes semigroup.
4.4 Endpoint estimates for the Stokes semigroup.
4.5 Application to the Navier-Stokes equations.

Course Description

The purpose of this course is to derive useful estimates for the Stokes semigroup by using real interpolation, which is one of real analytic tools indispenable in recent detailed analysis for the Navier-Stokes equations, We start with the definition and fundamental property of real interpolation, together with the Besov spaces and the Lorentz spaces as concrete examples. We next investigate the relationship between real interpolation and duality, which is one of the main methods in this lecture. Then we proceed to the $L^q - L^r$ estimates of the Stokes semigrop, an important tool in the analysis of the Navier-Stokes equations, and verify that the methods above can be applied also to the Lorentz spaces and yield precise estimates. We finally study of the application of these estimates to some concrete nonlinear problems.